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DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

AERONAUTICAL RESEARCH LABORATORY

MELBOURNE, VICTORIA

Guided Weapons Technical Memorandum 012

REVIEW OF TECHNIQUES FOR IN-FLIGHT TRANSFER ALIGNMENT

by

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D. BUCCO

Approved for public release

JUNE 1992

AR-006-607

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SUMMARY

This report presents methods for transfer alignment (TA) of inertial navigation systems (INS) which have been published in the accessible literature during the last three decades. Kalman filtering techniques based on linearised dynamics dominated in the literature of the subject. Methods of TA can be classified as angular rates, velocity or position matching. In each case a number of assumptions are made to ensure the validity of proposed technique. The accuracy of filters depends on the particular implementation viz. allowable manoeuvres and time of TA, microprocessor used, vibration environment, inclusion of wing flexure into the model, type of application under discussion (range of missile or time of flight), quality of output from inertial measurement units, etc. Authors briefly discuss methods of in-flight transfer alignment of INS taking into account these assumptions.

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1. INTRODUCTION

The process of aligning an inertial navigation system (INS) using the data supplied by another INS is called in the literature a transfer alignment. More strictly transfer alignment is defined as a process of estimating the relative angular difference between positions of two (or more) coordinate systems by measuring the position of each of the systems with respect to the two or more vectors common to all coordinate systems (see [1]) for the purpose of updating and calibration of guidance instruments.

In-flight transfer alignment (TA) is applicable in the following circumstances :

(1) correction of aircraft's inertial navigation system (INS) using external data (output of additional sensors, set of terrain reference points, or coupling with GPS receiver, that is, the case of aided INS),

(2) initialisation and calibration of a weapon's inertial measurement unit (IMU) using the data from the carrier's (aircraft or ship) guidance instruments, with additional assumptions on accuracy of the carrier's navigation instruments.

We could also envisage, as a separate case, correction of data from the weapon's gyros during the independent flight (after the separation from the carrier) using additional instruments (magnetometers, GPS receiver, Doppler radar), but as far as the methods are concerned it coincides with (1) and only the accuracy requirements for tactical applications may differ. Some authors have reported (cf. [2], [3]) on more recent and current research work aimed at finding an efficient solution to this problem. A brief review of literature on transfer alignment is given by Barlow [4].

Usually the more complex system of aircraft's inertial instruments is referred to in the literature as a "master" INS, while the set of (usually strapdown) inertial sensors on a missile is called a "slave" IMU.

In this report we will concentrate on methods of aligning an unaided set of inertial sensors with emphasis on low-cost strapdown guidance instruments applicable to gliding stand-off munitions.

All methods of TA described in the accessible literature are based on particular vector matching techniques (dependent on physical justification) and almost all use an algorithm for state identification of stochastic systems known as a Kalman filter (KF). The filter may be characterised as a minimum variance estimator and its implementation is based on an assumption that probabilistic descriptions of errors are available on-line during the estimation. Other techniques worth mentioning are the maximum likelihood method and stochastic or deterministic differential games. It seems that deterministic adaptive model following (often referred to as MRAC - a model reference adaptive control) with uncertainty parameters belonging to some compact set (no specification of stochastic behaviour of the noise is necessary in this case) may also be useful for our application. In this review we concentrate only on the "mainstream approach". Alternative methods applicable for such a particular identification problem as TA are briefly mentioned at the end of this report.

Prior to the description of methods for TA we briefly present basic formulae used for the estimation problem (words : *filtering, estimation, identification* are used here interchangeably) by Kalman filters

2. THE DESIGN OF A KALMAN FILTER

2.1 Linear estimation

In the general case the equations for a linear discrete Kalman filter are as follows. **The system model** is represented by N stochastic linear difference equations for the state vector x_k:

$$x_k = \Phi_{k-1}x_{k-1} + \Gamma_u u_{k-1} + \Gamma\vartheta_{k-1} \qquad \vartheta_k \sim N(0, Q_k) \tag{1}$$

where Φ is an $N \times N$ nonsingular transition matrix, Γ_u is an $N \times r$ input matrix with constant entries, u_k is an r-dimensional, deterministic (perfectly known) input (control), ϑ_k is an uncorrelated white noise, referred to as a process noise, while Γ ($n \times q$ constant diagonal matrix) determines the strength of the noise. In most of the discussed TA algorithms the term referring to deterministic input is ignored. Including it causes a minor change in the state propagation equation (formulae (5)-(5a)).

The measurement model is designed as a set of M equations

$$z_k = H_k x_k + \varrho_k \qquad \varrho_k \sim N(0, R_k) \tag{2}$$

with H being an $M \times N$ matrix and ϱ_k the white noise, referred to as a measurement noise.

Initial conditions are assumed to be known

$$E[x(0)] = x_0, \qquad E[(x(0) - x_0)(x(0) - x_0)^T] = P_0 \tag{3}$$

where P_0 is the initial covariance matrix giving a statistical measure of confidence that the states are error-free.

Other assumptions are concerned with the correlation of the noise

$$\begin{cases} E[\vartheta_k \varrho_j^T] = 0, & \text{for} \quad k = 1, 2, \ldots, N \quad \text{and} \quad j = 1, 2, \ldots M \\ E[\varrho_k \varrho_j^T] = R\delta_{kj} & \text{for} \quad j, k = 1, 2, \ldots, M \\ E[\vartheta_k \vartheta_j^T] = Q\delta_{kj} & \text{for} \quad k, j = 1, 2, \ldots, N \end{cases} \tag{4}$$

where δ_{kj} denotes Kronecker delta function.

The state estimates are then extrapolated

$$\hat{x}_k^- = \Phi_{k-1}\hat{x}_{k-1}^+ \tag{5}$$

to obtain the components of the state-vector at the beginning of the k-th interval using the values obtained at the end of the (k-1)-th interval. With the deterministic input present in the system model we would have

$$\hat{x}_k^- = \Phi_{k-1}\hat{x}_{k-1}^+ + \Gamma_u u_{k-1} \tag{5a}$$

Error covariance extrapolation is calculated via the matrix equation

$$P_k^- = \Phi_{k-1}P_{k-1}^+\Phi_{k-1}^T + \Gamma Q_{k-1}\Gamma \tag{6}$$

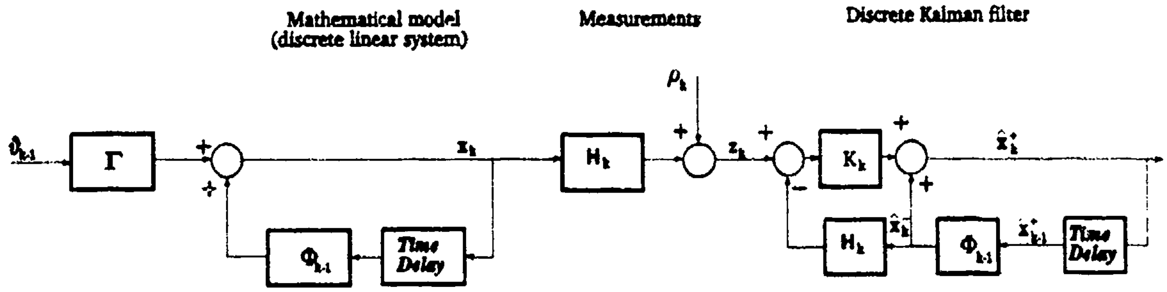

Figure 1 General Filter Configuration

The Kalman gain matrix may now be calculated as :

$$K_k = P_k^- H_k^T [H_k P_k^- H_k^T + R_k]^{-1} \tag{7}$$

which in turn is used to update the state estimates

$$\hat{x}_k^+ = \hat{x}_k^- + K_k [z_k - H_k \hat{x}_k^-] \tag{8}$$

and the error covariance matrix

$$P_k^+ = [I - K_k H_k] P_k^- = [I - K_k H_k] P_k^- [I - K_k H_k]^T + K_k R_k K_k^T \tag{9}$$

The whole process may be easily represented by the block-diagram shown in figure 1.

The above formulae are valid only if the specific assumptions (on the linearity of the process, white, uncorrelated noise and on initial conditions are fulfilled. In particular one must remember that the assumption on stochastic controllability and observability is also applied to (1)-(2), which means that (cf. [5] for formal definitions)

$$\begin{cases} \alpha I \le \displaystyle\sum_{j=i-N+1}^{i} \Phi^T(t_i, t_j) H^T(t_j) R^{-1}(t_j) H(t_j) \Phi(t_j, t_i) \le \beta I \\ \qquad\qquad\qquad \text{for stochastic observability} \\ \alpha I \le \displaystyle\sum_{j=i-N+1}^{i} \Phi(t_i, t_j) \Gamma(t_{j-1}) Q(t_{j-1}) \Gamma^T(t_{j-1}) \Phi^T(t_i, t_j) \le \beta I \\ \qquad\qquad\qquad \text{for complete stochastic controllability} \end{cases} \tag{10}$$

where $0 < \alpha < \beta < \infty$. When the above assumptions are satisfied the filter is uniformly asymptotically stable (cf. [5]). In the case when the system and/or measurement model are nonlinear - i.e. equations (1)-(2) are replaced by (11)-(12) - and either the linearised or extended Kalman Filter are used.

2.2 Linearised KF

The system is modelled now by a system of stochastic nonlinear differential equation, in particular for a continuous case

$$\dot{x}_i = f_i[\mathbf{x}(t), \mathbf{u}(t),\ t] + G_i \vartheta_i(t)\ , \tag{11}$$

with $\vartheta_i(\cdot)$ a white noise, $i = 1, 2, \ldots, N$, where N is the dimension of the process, G_i defines the strength of the noise and $\mathbf{f}(\cdot)$ fulfilling standard assumptions, so it is lipschitzian with respect to x_i and continuous w.r.t. control variables. Both $\mathbf{f}(\cdot)$ and $\mathbf{u}(t)$ are assumed to be piecewise continuous w.r.t. time. The measurements taken at time intervals (t_k, t_{k+1}) are modelled by the nonlinear function

$$\mathbf{z}(t_k) = \mathbf{h}[\mathbf{x}(t_k), t_k] + \varrho(t_k) \tag{12}$$

where $\varrho(\cdot)$ is defined in the same way as for the linear filter. It is further assumed that a nominal (reference) state trajectory $\mathbf{x}_{ref}$ can be generated in such a way that it satisfies the deterministic ordinary differential equation :

$$\dot{\mathbf{x}}_{ref}(t) = \mathbf{f}[\mathbf{x}_{ref}(t), \mathbf{u}(t), t] \tag{13}$$

It is assumed that the deviations from the reference trajectory are small enough to justify the use of linear perturbation technique. In order to satisfy this requirement the same control variables are used in nominal and real models. Denoting

$$\begin{aligned} \delta\mathbf{x} &= \mathbf{x} - \mathbf{x}_{ref} \qquad \text{and consequently} \\ \delta\dot{\mathbf{x}} &= \dot{\mathbf{x}} - \dot{\mathbf{x}}_{ref}(t) \end{aligned} \tag{14}$$

the process $\delta\dot{\mathbf{x}}$ satisfies the nonlinear stochastic differential equation

$$\delta\dot{\mathbf{x}} = f[\mathbf{x}, \mathbf{u}, t] - f[\mathbf{x}_{ref}, \mathbf{u}, t] + \mathbf{G}(t)\vartheta(t) \tag{15}$$

Expanding the RHS of the above equation as a Taylor series we have

$$\delta\mathbf{x} \frac{\partial\mathbf{f}[\mathbf{x}, \mathbf{u}(t), t]}{\partial\mathbf{x}}\bigg|_{\mathbf{x}=\mathbf{x}_{ref}(t)} + \text{H.O.T.} + \mathbf{G}(t)\vartheta(t) \tag{16}$$

After discarding higher order terms the original nonlinear equation has its first order approximation :

$$\delta\dot{\mathbf{x}} = \mathbf{F}[\mathbf{x}_{ref}(t), t]\delta\mathbf{x}(t) + \mathbf{G}(t)\vartheta(t) \tag{17}$$

where $\mathbf{F}(\cdot)$ is an n by n matrix with entries being partial derivatives of $f(\cdot)$ evaluated along the reference trajectory and $\mathbf{G}(t)$ is a diagonal matrix with entries $G_i(t)$. Similarly the perturbation measurement model of discrete observation is obtained

$$\delta\mathbf{z}(t_k) = \mathbf{H}[\mathbf{x}_{ref}(t_k), t_i]\delta\mathbf{x}(t_k) + \varrho(t_k) \tag{18}$$

where $\mathbf{H}$ is defined :

$$\mathbf{H} \triangleq \frac{\partial\mathbf{h}[\mathbf{x}, t_k]}{\partial\mathbf{x}}\bigg|_{\mathbf{x}=\mathbf{x}_{ref}(t_k)} \tag{19}$$

Converting equation (17) to discrete form and applying the previously defined **linear filter** (equations (7) and (8)) we obtain an optimal estimate of $\delta\mathbf{x}(t)$ denoted by $\widehat{\delta\mathbf{x}}(t)$. Then the best estimate of the total state is

$$\hat{\mathbf{x}}(t_k) = \mathbf{x}_{ref} + \widehat{\delta\mathbf{x}}(t_k) \tag{20}$$

The procedure ending in the above estimate is called a **linearised** Kalman filter. The assumption of small deviation from the reference trajectory is certainly the most limiting one and the next approach tries to overcome this shortcoming of the linearised filter.

2.3 Extended KF

For the extended KF the process of linearisation of state and measurement systems is repeated after each evaluation of $\hat{\mathbf{x}}$, with the initial condition defined as the recent update of the state.

As a result the formulae for gains, state estimate and covariance matrix of the extended filter may be expressed in the following way. The solution to the reference (nominal) equation with the initial condition $\hat{\mathbf{x}}(t_k^+)$ is denoted $\mathbf{x}_{ref}(t/t_k)$. Then

$$\begin{cases} \mathbf{K}(t_k) = \mathbf{P}(t_k^-)\mathbf{H}^T[t_k, \hat{\mathbf{x}}(t_k^-)]\{\mathbf{H}[t_k, \hat{\mathbf{x}}(t_k^-)]\mathbf{P}(t_k^-)\mathbf{H}^T[t_k, \hat{\mathbf{x}}(t_k^-)] + \mathbf{R}(t_k)\}^{-1} \\ \hat{\mathbf{x}}(t_k^+) = \hat{\mathbf{x}}(t_k^-) + \hat{\delta\mathbf{x}}(t_k^+) = \hat{\mathbf{x}}(t_k^-) + \mathbf{K}(t_k)\{\mathbf{z}_k - \mathbf{h}[\hat{\mathbf{x}}(t_k^-), t_k]\} \\ \mathbf{P}(t_k^+) = \mathbf{P}(t_k^-) - \mathbf{K}(t_k)\mathbf{H}[t_k, \hat{x}(t_k^-)]\mathbf{P}(t_k^-) \end{cases} \tag{21}$$

The estimate is propagated from t_k to t_{k+1} via integration of nominal differential equations with $\mathbf{x}_{ref}$ replaced by $\hat{\mathbf{x}}$. Matrices $\mathbf{F}$ and $\mathbf{H}$ are defined in a similar way as for the linearised filter except that the entries are now evaluated at each $\hat{\mathbf{x}}(t_k^-)$. Estimation of the covariance matrix explicitly depends on the estimate of the state variables.

2.4 Reducing the order of Kalman filter

We could deal here with two possible cases : either we design a system and observation model having a minimal number of variables and then use the standard algorithms described in 2.1-2.3, or if the information on a subset of variables of an existing model ceases to be important, or for some reason is unavailable, such variables may be excluded from further processing. The reduction by design depends on the physical relationship between variables, i.e. choice of particular equations for observation and system models. Examples of such filters applied to the transfer alignment problem are presented in 3.4. An exclusion of information on some subset of variables from further processing is called a reduced order KF. There is a number of suboptimal and optimal techniques dealing with the reduced order KF as discussed in [5] and [6].

One set of possible algorithms for optimal reduced order filtering is given by Simon and Stubberud [6]. The theoretical derivation is illustrated by an example of a hypothetical satellite carrying a strapped-down IMU and two star-trackers. Information from the star tracker is used to calibrate the IMU's gyro-biases, scale factors and misalignments.

This particular technique is formulated in the following way. The system model is assumed to consist of two blocks

$$x_{k+1} = \Phi_k x_k + \vartheta \tag{22}$$

$$\alpha_{k+1} = A_k x_k + E_k \alpha_k + s_k \tag{23}$$

while the observation model may be presented as

$$z_k = H_k x_k + L_k \alpha_k + \varrho_k \tag{24}$$

where the vectors α and s are p-dimensional, z and ϱ are m-dimensional, and the system and observation matrices are of appropriate dimensions. Furthermore the standard assumptions for ϑ, s and ϱ apply, that is they are uncorrelated white noise processes, with respective covariances denoted by R_k, S_k, and V_k. The initial conditions for the system model are the gaussian random variables with known means, say x_0, $\hat{\alpha}_0$, and covariances X_0 and Σ_0. Simon and Stubberud assume that $(L_k^T L_k)$ has an inverse and x_k is independent of α_k.

The estimates $\hat{\alpha}_k$ of α_k are not required in the process so the system may be reduced from (n+p) to (n). Instead of the output z_k the authors introduce the difference operator

$$y_{k+1} \triangleq z_{k+1} - L_{k+1} E_k L_k^* z_k \tag{25}$$

where L_k^* is a one-sided pseudo-inverse with assumption that it coincides with L_k^{-1} if the latter exists or

$$L_k^* = (L_k^T L_k)^{-1} L_k^T \tag{26}$$

The existence of L_k^* requires $p \leq m$ and that rank L_k=p. The new difference operator replaces the original observation model, so now

$$y_{k+1} = M_k x_k + w_{k+1} \tag{27}$$

with

$$M_k = H_{k+1}\Phi_k + L_{k+1} + L_{k+1}A_k - L_{k+1}E_k L_k^* H_k \tag{28}$$

$$w_{k+1} = H_{k+1}\vartheta_k + L_{k+1}s_k + \varrho_{k+1} - L_{k+1}E_k L_k^* \varrho_k \tag{29}$$

However the measurement noise is now coloured, so the standard assumptions of the basic Kalman filter no longer apply. The noise is correlated over one step with the covariance matrices specified as

$$E[w_{k+1}w_{k+1}^T] \equiv W_{k+1} = H_{k+1}R_k H_{k+1}^T + L_{k+1}S_k L_{k+1}^T + V_{k+1} + L_{k+1}E_k L_k^* V_k L_k^* E_k^T L_{k+1}^T \tag{30}$$

$$E[w_{k+1}w_k^T] \equiv W_{k+1,k} = -L_{k+1}E_k L_k^* V_k \tag{31}$$

$$E[w_{k+j}w_k^T] = 0 \qquad \text{for} \quad j > 1 \tag{32}$$

$$E[w_0 w_0^T] \equiv V_0 + L_0 \Sigma_0 L_0^T \tag{33}$$

$$E[w_{k+1}\vartheta_k^T] \equiv Q_k = H_{k+1}R_k \tag{34}$$

$$E[w_{k+1}\vartheta_{k+j}^T] = Q_k \delta_{j0} \tag{35}$$

$$E[\vartheta_j \vartheta_k^T] = R_k \delta_{jk} \tag{36}$$

Finally the reduced order model can be presented as :

$$x_{k+1} = \Phi_k x_k + \vartheta \tag{37}$$

$$y_{k+1} = M_k x_k + w_{k+1} \tag{38}$$

$$y_0 = H_0 x_0 + w_0 \tag{39}$$

with the initial conditions and covariances specified as above. The authors then obtained the minimum variance, unbiased estimate $\hat{x}_k$ of x_k based on measurement y_k, which is generated sequentially via the following equations :

$$P_{k+1} = M_k J_k M_k^T - W_{k+1,k} P_k^{-1} W_{k+1,k}^T - M_k C_k P_k^{-1} W_{k+1,k}^T$$
$$- W_{k+1,k} P_k^{-1} C_k^T M_k^T + W_{k+1} \tag{40}$$

$$C_{k+1} = \Phi_k J_k M_k^T - \Phi_k C_k P_k^{-1} W_{k+1,k}^T + Q_k^T \tag{41}$$

$$\Delta y_{k+1} \equiv y_{k+1} - \hat{y}_{k+1/k} = y_{k+1} - M_k \hat{x}_k - W_{k+1,k} P_k^{-1} \Delta y_k \tag{42}$$

where $\hat{y}_{k+1/k}$ is defined as the best estimate of y_{k+1} based on y_k.

$$\hat{x}_{k+1} = \Phi_k \hat{x}_k + C_{k+1} P_{k+1}^{-1} \Delta y_{k+1} \tag{43}$$

$$J_{k+1} = \Phi_k J_k \Phi_k^T + R_k - C_{k+1} P_{k+1}^{-1} C_{k+1}^T \tag{44}$$

where J_{k+1} is the estimate error covariance.

A different approach for model reduction is presented by Kortum in [7]. The method deals with the model reduction in the case when the system noise is either white or coloured, while the measurement noise has the classical white noise characteristics. In the case when the measurement noise is also coloured we may apply the Simon-Stubberud method. The procedure relies on splitting the n-dimensional state vector x into portions, say x_1 and x_2, of order n_1, n_2 respectively, so $n_1 + n_2 = n$ and $[x] = [x_1, x_2]$. The initial set of system equations may now be written as

$$\dot{x}_1 = \Phi_{11} x_1 + \Phi_{12} x_2 \tag{45}$$

$$\dot{x}_2 = f(x_1, x_2, u, \vartheta) \tag{46}$$

where u is an input (control) parameter, and ϑ an uncorrelated noise with power spectral density Q. The measurement equations have to be designed in the form :

$$z_1 = H_1 x_1 + \varrho_1 \tag{47}$$

$$z_2 = H_2 x_2 + \varrho_2 \tag{48}$$

It is further assumed that measurements z_2 are fairly accurate in order to provide good information on x_2, that the matrix H_2 is nonsingular and that ϱ_1 and ϱ_2 are white noise processes with the specified power spectral densities.

In terms of modelling navigational systems, Kortum suggests that equation (46) may represent a nonlinear, coupled dynamic equation of motion, while (45) represents the set of linearised kinematic relations including the angles and angular velocity components. It then follows from the assumptions on the observation mode that accurate knowledge of all dynamic state components is required. The values can be directly obtained from (48) and then substituted into (45) to yield

$$\dot{x}_1 = \Phi_{11} x_1 + \Phi_{12} H_2^{-1} z_2 - \Phi_{12} H_2^{-1} \varrho_2 \tag{49}$$

In view of (49) z_2 can now be interpreted as known input (control) parameter, and ϱ_2 becomes the process noise, which in turn may now be assumed to be coloured. In this case the state equations should be augmented to include

$$\dot{\varrho}_2 = \alpha_2 \varrho_2 + \vartheta \tag{50}$$

where ϑ is white noise. In the case when ϱ_1 becomes coloured noise, other special algorithms should be applied (cf. Maybeck [5]).

2.5 Compression of data

The problem of data averaging for TA has been considered by Powell and Bryson [8], and Bar-Shalom [9]. The idea is to preprocess the measurement data between successive updates of the Kalman filter. The filter's processing time is usually longer (especially in the case of full order estimation of gyro drifts, accelerometer biases, position, velocity and attitude errors) than the measurement interval. The compressed data would be otherwise excluded from the algorithm. The main motive for averaging is "to slow the filter update rate to save computation time" [8]. The technique is a trade-off between the total loss of information due to its exclusion and loss due to the averaging.

Loss of information should be kept low to minimise estimation error. Therefore the Brys Powell technique is applicable if :

1) entries of system and observation matrices (Φ and H) vary slowly

2) the process noise is weak by comparison to the measurement noise

3) a priori state information outweighs the new information in measurement batch.

The investigations in [8] indicated that, for the particular problem considered, up to 25 observations could be averaged with considerable computational savings and the most important factor behind the decision on the number of observations that could be averaged is the process-to-measurement-noise ratio.

2.6 Parallel processing of information

Another method of accommodating the computational requirement associated with often nonlinear and high-dimensional systems is to decentralize KF algorithm and allow parallel processing of information. Gardner [10] discussed an algorithm which processes outputs of Doppler radar and an inertial measurment system in order to estimate the misalignment of sensors. The method, called gain transfer algorithm, assumes processing information at local nodes (associated with the sensors) and then sending the estimates to central processor which obtains global estimates based on full system model. Another method is proposed by Rao and Durran-Whyte [11]. The fully decentralized algorithm totally eliminates the need for the hierarchical data processing. Local nodes form a communication network, sending their estimates to every other node and estimating some of the states of global model. Advantages of such an approach are presented in [11].

3. IN-FLIGHT TRANSFER ALIGNMENT ALGORITHMS

3.1 General assumptions

Full-order (all states) estimation algorithms give on-line information on sensor errors i.e. provide current knowledge on either **constant or dynamic** misalignment (attitude and heading compared with the carrier's INS or difference in Euler angles referred to the aircraft's axes) combined with the velocity errors and errors resulting from gyro drift rates and accelerometer biases. Assumption of constant misalignment angles excludes the effects of vibrations and flexibility and a KF based on this assumption may not be suitable for realistic application. However, for the sake of clarity, the presentation of transfer alignment methods starts from the simplest case based on rate matching, which in turn is augmented to more complicated cases. For full order modelling of an advanced Inertial Navigation System (as on the carrier aircraft) the following variables could be included :

- components of position error $r = [r_N,\ r_E,\ r_D]$ i.e with respect to North, East and vertical axes,

- components of static (constant) misalignment vector $\gamma = [\gamma_X,\ \gamma_Y,\ \gamma_Z]$, or $\gamma = [\gamma_N,\ \gamma_E,\ \gamma_D]$ depending on the reference axes,

- components of dynamic misalignment due to structural flexure (low-frequency elastic deformations) and vibrations (high frequency motions) $\eta(t) = [\eta_X(t),\ \eta_Y(t),\ \eta_Z(t)]$, so the total misalignment angle will be denoted by ζ where

$$\zeta \triangleq \gamma + \eta(t)$$

- vectors of angular rates of both "master" and "slave" guidance instruments (in case of angular rate matching) $\omega_m = [p, q, r]$ and $\omega_s = [p_s, q_s, r_s]$, and respective skew-symmetric matrices of angular rates denoted by $[\omega_m]$, $[\omega_s]$,

- components of velocity vectors (when appropriate) denoted by v and v_s for master INS and slave IMU respectively, or errors in North, East and down velocity components denoted by $\Delta v = [\Delta v_N,\ \Delta v_E,\ \Delta v_D]$

- components of specific force denoted by $f = [f_X,\ f_Y,\ f_Z]$ or $[f_N,\ f_E,\ f_D]$ depending on the reference axes, together with their corresponding skew-symmetric matrix denoted by [f].

- vector and skew-symmetric matrix of linear acceleration denoted by $a = [a_x,\ a_y,\ a_z]$ and $[a]$ respectively.

- gyro drifts and acceleration biases (notation ϵ, ∇ respectively with the components expressed in some chosen reference frame),

- gravity deflection and anomaly,

- altimeter output and bias,

- longitude and latitude angles denoted by λ and L, with $\delta\lambda$ and δL referring to the errors in respective quantities and sL, cL denoting sine and cosine of the angle .

- earth's radius and its angular rate denoted by R and Ω respectively.

The above notation tries to follow that of the standard textbooks (cf. Britting [12]). However, in some instances different symbols were introduced in order to unify the approach.

Stand-off gliding munitions usually require only very rudimentary navigation and control systems. In many instances the guidance system is reduced to either a set of gyros or to a relatively inexpensive AHRS (there is also an option of an aided navigation system, but it is outside the scope of this report). However, when the seeker is not locked onto the target before weapon's release, i.e. when the missile trajectory can be partitioned into the mid-course and terminal stages, the initialisation error together with the drifts and biases may cause the weapon to deviate from the planned path, which in turn may cause its failure to acquire the target.

Control of the roll and pitch angle in the mid-course guidance stage, together with bank to turn manoeuvres in the terminal phase introduces the requirement for relatively accurate information about rates, angles (attitude control) and accelerations to be supplied to the autopilot. Therefore only a subset of the listed set of variables need to be used in the KF models designed for aligning the weapon's AHRS prior to its mission. A minimum set of variables should include the attitude and azimuth misalignment and some modelling of flexural motion. Additionally one can also include models for gyro drifts and accelerometer biases, but such an inclusion depends on time allowed for the TA.

In our analysis we shall make the following standard assumptions :

1) no equipment failure,

2) compatibility between TA and other functions sharing the same microprocessor,

3) master INS drift free, slave INS driven by relative error,

4) rigid or flexible structure of aircraft (different design of filter results from a particular assumption),

5) data-latency problem solved (synchronisation of the outputs from IMU with the output from the aircraft),

6) exact collocation between seekers reference axis and INS coordinates (i.e. coincidence of coordinate axis direction).

The last assumption is not strictly required for the following analysis, but we include it in the above list to signal the possible problem associated with the terminal phase of guidance on some classes of weapons.

After rough initialisation of the missile's IMU there is a choice of three fundamental procedures (or rather sets of procedures) for the transfer alignment, namely position matching, angular rate matching and velocity matching. These procedures may be considered with respect to either a rigid or a flexible structure, with or without modelling of drifts and biases, and applying any of the presented filtering techniques. The basics of the procedures for TA are discussed in 3.2-3.5.

An unaided attitude and heading reference system could be also "roughly" initialised by using the attitude (angles only) data supplied by the aircraft INS. This however is not a case for "transfer alignment" (as position, velocity or angular rates errors are not estimated), but what could be called an attitude alignment with "coarse" initialisation via a statistical averaging procedure. For the proper transfer alignment the required structure of information is discussed in the following paragraphs.

3.2 Angular rate matching

Angular rate matching is a conceptually simple, but relatively inaccurate method for estimating the angular errors between two coordinate systems by using a suitable physical relationship between their angular rates and the misalignment. The procedure for estimation of the misalignment follows that described by Schneider [13]. The usage of the classical Kalman filter is justified by a formulation which relates **linearly** the misalignment angles to the difference in the angular velocity of master and slave gyros (it is based on an orthogonality assumpion which in turn relies on frequent updates of data from the aircraft).

The measurement vector is formed from the differences between the master and slave INS's rates so using the previously introduced notation we have :

$$z_k \triangleq \Delta\omega_k = \begin{bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{bmatrix}_k = \omega_{m_k} - \omega_{s_k} = \begin{bmatrix} p - p_s \\ q - q_s \\ r - r_s \end{bmatrix}_k \tag{51}$$

The state vector x_k is the unknown (estimated) misalignment angle between the two reference frames (denoted by γ) :

$$x_k = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_k \triangleq \gamma = \begin{bmatrix} \gamma_X \\ \gamma_Y \\ \gamma_Z \end{bmatrix}_k = \begin{bmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{bmatrix}_k \tag{52}$$

It is assumed that the aircraft structure is rigid, i.e. we deal with the case of **constant** misalignment (see Fig. 2). Then $\Phi_{k-1} \equiv I_3, \forall k \in N$. The system model is therefore

$$x_k = x_{k-1} + \vartheta_{k-1} \tag{53}$$

where ϑ_k is modelled by the Gaussian white noise of variance Q_k. Normally for implementation purposes ϑ would represent a white noise with spectral density taken as

$$Q_k = \alpha \Delta t_k I_3 \tag{54}$$

where $\Delta t_k = t_k - t_{k-1}$ and α a suitable constant to introduce the noise. The choice of α is a real problem, since it determines the speed of convergence via covariance matrix updates.

Under the suitable assumptions that outputs of gyros in both INS's are known and synchronised and Δt is small enough so the angular velocity vector does not change direction (at t_k as compared to t_{k-1}) "appreciably", the following algorithm can be derived.

In angular rate matching the observation (measurement) equation is based on the following kinematic relationship (see Schneider op. cit.) :

$$z_k = \gamma \times \omega \tag{55}$$

where the second term of the cross product refers to the aircraft's p,q, and r. After some algebraic manipulations it reduces to :

$$z_k = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix}_k x_k + \begin{bmatrix} \varrho_1 \\ \varrho_2 \\ \varrho_3 \end{bmatrix}_k \tag{56}$$

Thus the observation matrix H_k is now being defined as

$$H_k \equiv \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} \tag{57}$$

Full formulation of the filter would also incorporate the white noise ϱ_k characterised by spectral density R_k. This is introduced by Schneider as a diagonal matrix with entries defined as twice the variances of the measurement error in each gyro output denoted σ_ϑ^2.

In the explicit form we have the following set of equations for the KF. Since the Φ_k becomes the identity matrix, the state equations assume the form

$$x_k = \begin{bmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{bmatrix}_k = \begin{bmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{bmatrix}_{k-1} + \begin{bmatrix} \vartheta_1 \\ \vartheta_2 \\ \vartheta_3 \end{bmatrix}_{k-1} \tag{58}$$

with ϑ_k the white noise with previously defined spectral density Q_k. The measurement model (with the differences of angular rates, i.e. differences between outputs of gyros, being the measurable quantities) is now

$$z_k = \begin{bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{bmatrix}_k = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix}_k \begin{bmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{bmatrix}_k + \begin{bmatrix} \varrho_1 \\ \varrho_2 \\ \varrho_3 \end{bmatrix}_k \tag{59}$$

where ϱ_k has the spectral density defined as $R_k = 2\sigma_\vartheta^2 I_3$ (I_3 is 3×3 identity matrix).

The Kalman filter is implemented as follows. With $\hat{x}_0^+$, P_0^+ well defined initial conditions, the state estimates and the covariance matrix can now be extrapolated

$$\begin{bmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{bmatrix}_k^- = \begin{bmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{bmatrix}_{k-1}^+ \tag{60}$$

$$P_k^- = P_{k-1}^+ + Q_{k-1} = P_{k-1}^+ + \alpha\Delta t_{k-1} I_3 \tag{61}$$

and subsequently, Kalman gains matrix may now be calculated

$$K_k = P_k^- \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}_k \left\{ \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix}_k P_k^- \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}_k + \begin{bmatrix} 2\sigma_w^2 & 0 & 0 \\ 0 & 2\sigma_w^2 & 0 \\ 0 & 0 & 2\sigma_w^2 \end{bmatrix}_k \right\}^{-1} \tag{62}$$

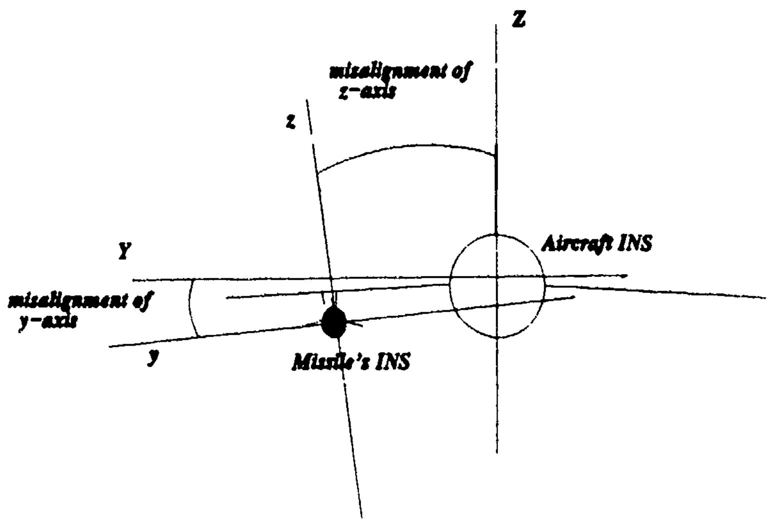

Figure 2 Misalignment for rigid structure.

The matrix K_k is now used to obtain the updates of state estimates ($\hat{x}_k^+$) and covariance matrix (P_k^+)

$$\begin{bmatrix} \hat{\Delta\phi} \\ \hat{\Delta\theta} \\ \hat{\Delta\psi} \end{bmatrix}_k^+ = \begin{bmatrix} \hat{\Delta\phi} \\ \hat{\Delta\theta} \\ \hat{\Delta\psi} \end{bmatrix}_k^- + K_k \left[\begin{bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{bmatrix}_k - \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix}_k \begin{bmatrix} \hat{\Delta\phi} \\ \hat{\Delta\theta} \\ \hat{\Delta\psi} \end{bmatrix}_k^- \right] \tag{63}$$

$$P_k^+ = \left[I_3 - K_k \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix}_k \right] P_k^- \tag{64}$$

which are the starting values for the new cycle of computations.

3.2.1 Inclusion of flexibility and vibrations into the algorithm

Augmenting Schneider's algorithm presented in section 3.2 as it was originally derived in [13], we have now the total misalignment angle defined as $\zeta \triangleq \gamma + \eta(t)$ where γ refers to the static misalignment of the rigid structure while components of $\eta(t)$ define dynamic bending of aircraft structure. Since the bending is due to the stochastic factors the time history of η may be modelled with the second order Markov processes, while $\dot\gamma \equiv 0$. The vector of state variables x will consist of 9 components, the first three being the static misalignment and the other 6 describing the time dependent flexure angles along three axes. The problem of dynamic misalignment is presented graphically in Figure 3.

It is assumed that each equation fo. bending angle may be written as

$$\ddot\eta_i + 2\xi_i\dot\eta_i + \xi_i^2\eta = \vartheta_i \tag{65}$$

where index i refers to a particular axis. The equation as such describes the behaviour of a second order random process excited by a white noise input. Switching to the system of first-order equation the vector of state variables is formed

$$\mathbf{x}^T = [x_1, x_2, \ldots, x_9]^T = [\gamma_X, \gamma_Y, \gamma_Z, \eta_X, \eta_Y, \eta_Z, \dot\eta_X, \dot\eta_Y, \dot\eta_Z]^T \tag{66}$$

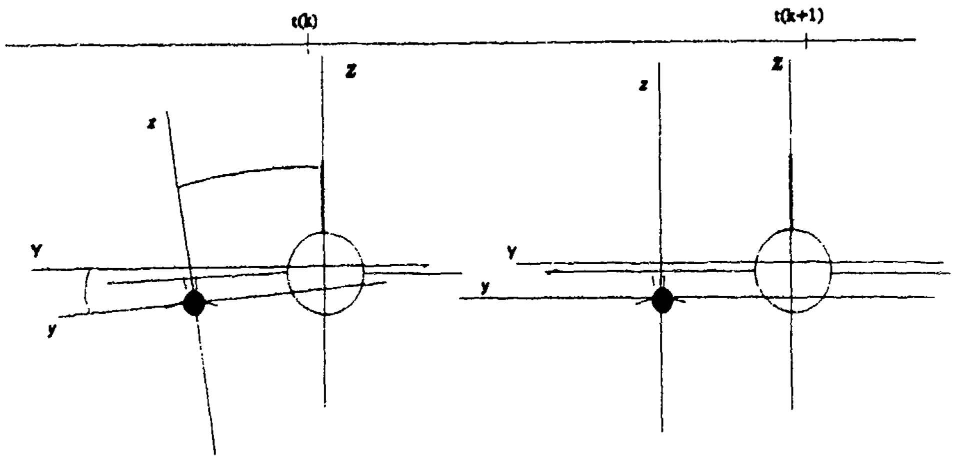

Figure 3 Dynamic misalignment.

so now we can set up the continuous-state system model :

$$\dot{\mathbf{x}} = \Phi \mathbf{x} + \vartheta \tag{67}$$

or in the explicit form

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\xi_X^2 & 0 & 0 & -2\xi_X & 0 & 0 \\ 0 & 0 & 0 & 0 & -\xi_Y^2 & 0 & 0 & -2\xi_Y & 0 \\ 0 & 0 & 0 & 0 & 0 & -\xi_Z^2 & 0 & 0 & -2\xi_Z \end{bmatrix} \begin{bmatrix} \gamma_X \\ \gamma_Y \\ \gamma_Z \\ \eta_X \\ \eta_Y \\ \eta_Z \\ \dot{\eta}_X \\ \dot{\eta}_Y \\ \dot{\eta}_Z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vartheta_X \\ \vartheta_Y \\ \vartheta_Z \end{bmatrix} \tag{68}$$

The observation model (still linear !) will be based on the following relationship :

$$\mathbf{z} = \zeta \times \omega - \dot{\eta} \tag{69}$$

or in matrix form

$$\begin{bmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{bmatrix} = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} \begin{bmatrix} \gamma_X + \eta_X \\ \gamma_Y + \eta_Y \\ \gamma_Z + \eta_Z \end{bmatrix} - \begin{bmatrix} \dot{\eta}_X \\ \dot{\eta}_Y \\ \dot{\eta}_Z \end{bmatrix} \tag{70}$$

Using state-space form required by KF we have for the measurement model

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 & r & -q & 0 & r & -q & -1 & 0 & 0 \\ -r & 0 & p & -r & 0 & p & 0 & -1 & 0 \\ q & -p & 0 & q & -p & 0 & 0 & 0 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \varrho_1 \\ \varrho_2 \\ \varrho_3 \end{bmatrix} \tag{71}$$

The discretisation of the model follows from the relation

$$\Phi_k = I_9 + \Phi \Delta t_k \tag{72}$$

which is valid provided Δt_k is much smaller than the shortest period of the natural modes of the continuous system (Gelb [14]). Under this condition the discretised state equations assume the form defined by (1) with $\Gamma = I_9$, namely :

$$[\gamma_X \quad \gamma_Y \quad \gamma_Z \quad \eta_X \quad \eta_Y \quad \eta_Z \quad \dot{\eta}_X \quad \dot{\eta}_Y \quad \dot{\eta}_Z]_{k+1}^T =$$

$$\left[\begin{array}{ccccccccc} I_3 & & 0_3 & & & & 0_3 & & \\ 0_3 & & I_3 & & & & I_3\Delta t_k & & \\ & -\xi_X^2\Delta t_k & 0 & 0 & 1-2\xi_X\Delta t_k & & 0 & & 0 \\ 0_3 & 0 & -\xi_Y^2\Delta t_k & 0 & 0 & & 1-2\xi_Y\Delta t_k & & 0 \\ & 0 & 0 & -\xi_Z^2\Delta t_k & 0 & & 0 & & 1-2\xi_Z\Delta t_k \end{array}\right] \begin{bmatrix} \gamma_X \\ \gamma_Y \\ \gamma_Z \\ \eta_X \\ \eta_Y \\ \eta_Z \\ \dot{\eta}_X \\ \dot{\eta}_Y \\ \dot{\eta}_Z \end{bmatrix}_k \tag{73}$$

$$+ [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \vartheta_X \quad \vartheta_Y \quad \vartheta_Z]_k^T$$

The state estimates are extrapolated using formula (5) and are augmented versions of equations given explicitly for the case of a rigid structure. Equations (6) can be directly used for extrapolation of the covariance, where Q_{k-1} is now a diagonal matrix with elements of the main diagonal defined as

$$[\alpha\Delta t_{k-1},\ \alpha\Delta t_{k-1},\ \alpha\Delta t_{k-1},\ 0,\ 0,\ 0,\ 4\beta_X^3\sigma_X^2\Delta t_k,\ 4\beta_Y^3\sigma_Y^2\Delta t_k,\ 4\beta_Z^3\sigma_Z^2\Delta t_k,\ ] \tag{74}$$

3.2.2 *Specific forces (accelerations) matching for the measurement model*

The measurement models of KF for angular rates matching may also be augmented with the specific forces, if the outputs of accelerometers are available for processing. Schneider denotes the difference in specific force measurements by $z_{a,k}$ and then forms the suitable measurement model :

$$z_{a,k} \triangleq \Delta f = \begin{bmatrix} \Delta f_X \\ \Delta f_Y \\ \Delta f_Z \end{bmatrix} = \begin{bmatrix} f_X \\ f_Y \\ f_Z \end{bmatrix}_k - \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}_k \tag{75}$$

The observation model for accelerometers is based on the relation (see Schneider op. cit.)

$$\Delta f = \zeta \times (f + C) \tag{76}$$

where C is the contribution of the Coriolis force discussed later. In more convenient matrix form the previous equation may be written as :

$$\Delta f = ([f] + [C])\zeta \tag{77}$$

In the case of a rigid structure and when the Coriolis forces are negligible, the above two formulae may be reduced to

$$\Delta f = \gamma \times f \tag{76a}$$

$$\Delta f = [f]\gamma \tag{77a}$$

where matrix $[f]$ is a skew-symmetric matrix and relations between the vector f and entries of the matrix $[f]$ are the same as between ω and $[\omega]$, while the term C represents the correction due to the centripetal forces experienced by the missile's IMU and not present in outputs from the aircraft. Although not given explicitly here, entries of C are functions of the distance between the carrier/aircraft's CG point and the missile's CG and the angular velocity as sensed by the INS. It is assumed here that the master INS is situated ideally at the aircraft's CG while the slave IMU is located at the missile's CG. Giving the augmented observation model in the explicit form we have

$$z_k = \begin{bmatrix} \Delta p \\ \Delta q \\ \Delta r \\ \Delta f_X \\ \Delta f_Y \\ \Delta f_Z \end{bmatrix}_k = \left(\begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \\ 0 & f_Z & -f_Y \\ -f_Z & 0 & f_X \\ f_Y & -f_X & 0 \end{bmatrix}_k + [C]_k \right) \begin{bmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{bmatrix}_k + \begin{bmatrix} \varrho_1 \\ \varrho_2 \\ \varrho_3 \\ \varrho_4 \\ \varrho_5 \\ \varrho_6 \end{bmatrix}_k \tag{78}$$

Since the same state variables are used in the basic observation model of accelerometers, instead of matrix augmentation, two separate measurement models could be used (sequentially) as originally suggested in [13]. In the case when flexibility is taken into account we have the following measurement equation :

$$\begin{bmatrix} \Delta p \\ \Delta q \\ \Delta r \\ \Delta f_X \\ \Delta f_Y \\ \Delta f_Z \end{bmatrix}_k = \left(\begin{bmatrix} 0 & r & -q & 0 & r & -q & -1 & 0 & 0 \\ -r & 0 & p & -r & 0 & p & 0 & -1 & 0 \\ q & -p & 0 & q & -p & 0 & 0 & 0 & -1 \\ 0 & f_Z & -f_Y & 0 & f_Z & -f_Y & 0 & 0 & 0 \\ -f_Z & 0 & f_X & -f_Z & 0 & f_X & 0 & 0 & 0 \\ f_Y & -f_X & 0 & f_Y & -f_X & 0 & 0 & 0 & 0 \end{bmatrix}_k + [C(\eta,\dot{\eta},\omega)]_k \right) \begin{bmatrix} \gamma_X \\ \gamma_Y \\ \gamma_Z \\ \eta_X \\ \eta_Y \\ \eta_Z \\ \dot{\eta}_X \\ \dot{\eta}_Y \\ \dot{\eta}_Z \end{bmatrix}_k$$

$$+ [\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5, \varrho_6]^T{}_k \tag{79}$$

Again here we could consider two separate observation models for the sake of computational efficiency.

Under certain conditions the inclusion of additional variables in the state model will not contribute to the accuracy of the alignment process. Before augmenting the dimension of the filter to include gyro drift rates (either of the missile IMU or of both inertial systems) analysis of conditions affecting the process is necessary. For the clear-cut cases the conclusion is (see [1], [11]) that if requirements for the final accuracy of alignment are not too stringent, the carrier-aircraft executes some manoeuvres during TA resulting in large angular velocities, the time allowed for TA is relatively short, and the drift rates are low, then the inclusion of the gyro drift rates to the state model is not appropriate for low-cost AHRS.

The other extreme situation is when drift rates are high, duration of the alignment process is in order of 1 hour, high accuracy of the alignment is demanded, and the input of angular rates is relatively low, then the KF should include all equations modelling the gyro drift rates. As far as the other cases are concerned detailed simulations of a number of models is needed before the "optimal" (in terms of the speed and accuracy) set of variables entering the state model can be chosen.

Further augmentation of the filter to include drifts and biases, as well as the detailed algorithms for the explicit calculation of each entry constituting matrices Q and R is also contained in [13].

3.2.3 Low order algorithm for angular rates and specific forces matching

Another algorithm based on rate matching is given by Harris and Wakefield [15]. Instead of considering static misalignment γ and dynamic component η separately, they designed the models for the total misalignment angle ζ, thus limiting the system model to 3 first-order ordinary differential equations. In particular they considered the angular rate vectors in the "master" and "slave" frames ω_m, ω_s, and determined the time-derivative of total misalignment angle :

$$\dot{\zeta} = \omega_s - T\omega_m \tag{80}$$

then proceeded with the approximation of the direction cosines matrix taking

$$T(\zeta) = I + M(\zeta) \tag{81}$$

where M is a skew-symmetric matrix. The above formula is valid only for small misalignment angles. Substituting (81) into (80) using the relationship $M(\omega)\zeta = -M(\zeta)\omega$ and noting the difference between the actual and measured quantities gives the basic system model of KF

$$\dot{\zeta} = M(\omega_m^*)\zeta + \Delta\omega^* + \Delta\epsilon + \Delta v \tag{82}$$

where $(*)$ refers to the measured quantities, $M(\omega_m^*)$ coincides with the previously defined matrix $[\omega_m^*]$, $\Delta\epsilon$ denotes the difference between master and slave gyro drifts, while the last term refers to the difference in gyro quantisation noise.

The above model is then implemented in the KF structure by treating $\Delta\omega^*$ as a known forcing function and assuming that combined differences between the gyro drifts and quantisation noise have Gaussian characteristics.

As a measurement model acceleration and/or angular rate matching vectors are proposed. Summarising, the above model could be presented in the form of a discrete KF in the following way. The system model is taken as :

$$\begin{bmatrix} \zeta_x \\ \zeta_y \\ \zeta_z \end{bmatrix}_k = \begin{bmatrix} 1 & r\Delta t_k & -q\Delta t_k \\ -r\Delta t_k & 1 & p\Delta t_k \\ q\Delta t_k & -p\Delta t_k & 1 \end{bmatrix}_{k-1} \begin{bmatrix} \zeta_x \\ \zeta_y \\ \zeta_z \end{bmatrix}_{k-1} + \begin{bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{bmatrix}_{k-1} \Delta t_k + \begin{bmatrix} \vartheta_x \\ \vartheta_y \\ \vartheta_z \end{bmatrix}_{k-1} \tag{83}$$

The suggested observation (measurement) model would be either acceleration (or rather specific forces) matching or angular rates matching. In the first case we have

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}_k \triangleq \begin{bmatrix} \Delta f_x \\ \Delta f_y \\ \Delta f_z \end{bmatrix}_k = \begin{bmatrix} 0 & f_Z & -f_Y \\ -f_Z & 0 & f_X \\ f_Y & -f_X & 0 \end{bmatrix}_k \begin{bmatrix} \zeta_x \\ \zeta_y \\ \zeta_z \end{bmatrix}_k + \begin{bmatrix} \varrho_x \\ \varrho_y \\ \varrho_z \end{bmatrix}_k \tag{84}$$

while for the second case the discussed observation equation would be similar to the first of Schneider's model presented in 3.2 (except that the state vector γ is replaced by ζ). When specific force matching is used for the above measurement model, we require total information to be available from the master's and slave's accelerometers and gyroscopic instruments, which may be too rigorous a requirement for practical implementations. When angular rates matching is used, only gyros outputs are needed for the algorithm. For the former case the following equations form the KF algorithm. The state extrapolation is expressed by

$$\begin{bmatrix} \hat{\zeta}_x \\ \hat{\zeta}_y \\ \hat{\zeta}_z \end{bmatrix}_k^- = \begin{bmatrix} 1 & r\Delta t_k & -q\Delta t_k \\ -r\Delta t_k & 1 & p\Delta t_k \\ q\Delta t_k & -p\Delta t_k & 1 \end{bmatrix}_{k-1} \begin{bmatrix} \hat{\zeta}_x \\ \hat{\zeta}_y \\ \hat{\zeta}_z \end{bmatrix}_{k-1}^+ + \begin{bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{bmatrix}_{k-1} \Delta t_k \tag{85}$$

Error covariance matrix is extrapolated via

$$P_k^- = \begin{bmatrix} 1 & r\Delta t_k & -q\Delta t_k \\ -r\Delta t_k & 1 & p\Delta t_k \\ q\Delta t_k & -p\Delta t_k & 1 \end{bmatrix}_k P_{k-1}^+ \begin{bmatrix} 1 & r\Delta t_k & -q\Delta t_k \\ -r\Delta t_k & 1 & p\Delta t_k \\ q\Delta t_k & -p\Delta t_k & 1 \end{bmatrix}_k^T + Q_{k-1} \tag{86}$$

The Kalman gain matrix is now determined by

$$K_k = P_k^- \begin{bmatrix} 0 & f_Z & -f_Y \\ -f_Z & 0 & f_X \\ f_Y & -f_X & 0 \end{bmatrix}_k^T \left\{ \begin{bmatrix} 0 & f_Z & -f_Y \\ -f_Z & 0 & f_X \\ f_Y & -f_X & 0 \end{bmatrix}_k P_k^- \begin{bmatrix} 0 & f_Z & -f_Y \\ -f_Z & 0 & f_X \\ f_Y & -f_X & 0 \end{bmatrix}_k^T + \begin{bmatrix} 2\sigma_w^2 & 0 & 0 \\ 0 & 2\sigma_w^2 & 0 \\ 0 & 0 & 2\sigma_w^2 \end{bmatrix}_k \right\}^{-1} \tag{87}$$

where the matrix R_k (the last term in the bracket) was assumed to be the same as in (62). Then the state estimates and error covariance matrix updates are given by

$$\begin{bmatrix} \hat{\zeta}_x \\ \hat{\zeta}_y \\ \hat{\zeta}_z \end{bmatrix}_k^+ = \begin{bmatrix} \hat{\zeta}_x \\ \hat{\zeta}_y \\ \hat{\zeta}_z \end{bmatrix}_k^- + K_k \left[\begin{bmatrix} \Delta f_x \\ \Delta f_y \\ \Delta f_z \end{bmatrix}_k - \begin{bmatrix} 0 & f_Z & -f_Y \\ -f_Z & 0 & f_X \\ f_Y & -f_X & 0 \end{bmatrix}_k \begin{bmatrix} \hat{\zeta}_x \\ \hat{\zeta}_y \\ \hat{\zeta}_z \end{bmatrix}_k^- \right] \tag{88}$$

$$P_k^+ = \left[I_3 - K_k \begin{bmatrix} 0 & f_Z & -f_Y \\ -f_Z & 0 & f_X \\ f_Y & -f_X & 0 \end{bmatrix}_k \right] P_k^- \tag{89}$$

The augmentation of the original set of equations (83) follows from the inclusion of $\Delta\omega$ to the system model (as a part of the state variables vector). The dynamics of $\Delta\omega$ is modelled by

$$\Delta\dot{\omega} = -[\dot{\omega}_m^*]\zeta - [\omega_m^*]^2\zeta - [\omega_m^*]\Delta\omega + \Delta\dot{\omega}_f + \Delta\dot{\omega}_v \tag{90}$$

where subscripts f and v refer to low-frequency flexure and vibratory motions. The measurement model is formed from the angular rate matching vector and is given in [38] as

$$z = \Delta\omega^* = \Delta\omega + \varrho \tag{91}$$

The above is a short-cut version of the formal measurement model relating the observed quantities to the state vector $[\zeta, \Delta\omega]$, as in this case

$$\Delta\omega_k^* = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \zeta_x \\ \zeta_y \\ \zeta_z \\ \Delta p \\ \Delta q \\ \Delta r \end{bmatrix}_k + \begin{bmatrix} \varrho_x \\ \varrho_y \\ \varrho_z \end{bmatrix}_k \tag{92}$$

Finally we have a case when the system noise is assumed to be coloured. The system model has to be now augmented to include the set of equations

$$\dot{\vartheta}_i(t) = \alpha_i(t) \qquad i = 1, 2, \; \ldots \; , 6 \tag{93}$$

where $\alpha(t)$ is now assumed to be a white noise process. The vector of state variables will be in this case

$$x^T = [\zeta^T, \; \Delta\omega^T, \; \vartheta^T] \tag{94}$$

Harris and Wakefield also give reasons why, in theory, a low-dimensional filter may outperform (under certain circumstances) the augmented version. Firstly, the augmented filter is based on less accurate modelling and the measurement of $\Delta\omega$ via angular rate matching can produce greater uncertainty in the misalignment-angle estimate. The low-order (3-state) model is based on the measurements containing the vibration- and flexural noise and some of these uncertainties (viz. vibrations) should be removed via direct incorporation of measurements into the process (assuming good-quality gyros). Secondly, when the filter is run at the update rate of the order of the maximum vibratory frequency and additionally assuming that measurement of angular velocities ω_m, ω_s are incoming at higher rates then the integration of the lower-order model, using the measured angular rates may remove the uncertainty at the update times to a greater degree than could be achieved with the high-dimensional model.

Additionally, adaptive estimation of entries of the covariance matrix is better suited for low-order models.

Harris and Wakefield's approach is similar to one presented earlier by Schultz and Keyes [16] (and also to the results contained in [17]) in which acceleration and angular rates matching were used to align a missile's strapdown inertial sensors to the aircraft reference frame.

Schultz and Keyes presented results of the simulation based on two measurement models : acceleration matching only, and acceleration combined with rate matching. Their simulation results have shown that KF with combined measurement model requires about 3 seconds for alignment during a 3-g lateral S-shape manoeuvre and about 7 seconds for a straight and level flight with a small oscillatory roll, while the KF using acceleration matching only has achieved an alignment in about 15 seconds during the 3-g lateral manoeuvre and does not provide sufficient alignment capability for any low-g manoeuvre profiles.

3.2.4 Case of on-ground alignment

An angular rate matching technique has been also used for on-ground alignment in an approach given by Kortum [7]. The problem was concerned with alignment of an uncalibrated INS at rest. The basic model is formed by three equations being the linearised kinematic relation for the platform angles. Denoting γ_E, γ_N and γ_D the platform angles about East-West, North-South and the vertical axes respectively we have :

$$\begin{bmatrix} \dot{\gamma}_E \\ \dot{\gamma}_N \\ \dot{\gamma}_D \end{bmatrix} = \begin{bmatrix} 0 & \Omega_v & -\Omega_h \\ -\Omega_v & 0 & 0 \\ \Omega_h & 0 & 0 \end{bmatrix} \begin{bmatrix} \gamma_E \\ \gamma_N \\ \gamma_D \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{95}$$

The rates p,q, and r are available from the gyros. Since the gyros outputs are corrupted by noise (drifts), the equations for system model now include all components of (1). The measurment model uses the fact that the outputs of accelerometers mounted on the horizontal plane should be proportional to misalignment angles γ_E, γ_N.

Since we are concerned mainly with in-flight alignment, the detailed discussion of results contained in [7] lies outside the scope of this report. However the method used for the order reduction and some results concerning the modelling of gyro drifts and accelerometers biases are included in the following paragraphs.

3.3 Modelling of gyro drifts and accelerometer biases.

The modelling of drifts and biases of inertial instruments needs to take into account the type of hardware and the environment it is applied to (viz. gimballed and strapdown INS). Since in this review we do not consider a particular application of the transfer alignment algorithms, the ways drifts could be accounted for in the filter will consequently vary. We discuss here a number of models, but the choice of a particular one for implementation depends on the type of hardware used.

The first of the presented models was given by Baziw and Leondes in [18]. The gyro drifts rate is modelled as

$$\epsilon_i = \epsilon_{0_i} + [K_{\omega_i}, \gamma_{S_i}, \gamma_{o_i}]\omega + [K_{I_i}, K_{S_i}, 0]a_T + \omega^*[K^2_{\omega_i}]\omega + a_T^*[K^{2'}_{g_i}]a_T \tag{96}$$

while the model for i-th accelerometer error is

$$n_{am_i} = \nabla_i + [K_{a_i}, \alpha_{N_i}, \alpha_{C_i}]a_T + a_T^*[K^2_{g_i}]a_T \tag{97}$$

where the following notation introduced in the Baziw and Leondes' paper is used

∇_i is the bias of the i-th accelerometer.

a_T is the thrust acceleration in the i-th accelerometer (gyro) coordinates.

K_{a_i} is the scale factor of the i-th accelerometer.

α_{N_i} is the misalignment of the i-th accelerometer's input axis in the plane of the input and normal axes.

α_{C_i} is the misalignment of the i-th accelerometer's input axis in the plane of the input and cross axes.

K_{ω_i} is the scale factor of the i-th gyro.

γ_{S_i} (γ_{o_i}) is the misalignment of the i-th gyro input axis in the plane defined by input and the spin (output) axes.

K_{I_i} (K_{S_i}) is the i-th gyro error coefficient due to mass unbalance along the input (spin) axis.

$[K^2_{g_i}]$ is a symmetric matrix where the elements on the main diagonal represent accelerometer's nonlinearity along the i-th axis,

the entries of K^i_{jk} are the cross-axes sensitivity of the i-th accelerometer in the j-k plane.

$[K^2_{\omega_i}]$ is also a symmetric matrix where $K^i_{\omega_{jk}}$ are the error coefficients sensitive to the angular velocity components $\omega_j\omega_k$ for the i-th gyro.

$[K^{2'}_{g_i}]$ is an asymmetric matrix whose elements are the error coefficients sensitive to the acceleration components $a_j a_k$ (anisoelastic effect) for the i-th gyro.

Kortum [7] presented a particular case for modelling of the drift of gyroscopic instruments by interpreting it as Gauss-Markov process. The model was given in following form :

$$\hat{\epsilon} = \epsilon + d^r + d^c + \vartheta \tag{98}$$

where $\hat{\epsilon}$ is total drift, ϵ is a bias term, d^r is a random ramp, d^c is a correlated noise, and ϑ is, as previously defined, a white noise. In the form of state equations (98) can be presented as:

$$\dot{\epsilon} = 0 \tag{99}$$

$$\dot{d}^r = c^r \tag{100}$$

$$\dot{c}^r = 0 \tag{101}$$

$$\dot{d}^c = T_d^{-1} d^c + \vartheta^c \tag{102}$$

where T_d is the correlation time constant, c^r is the slope of the ramp and ϑ^c is a white noise. For practical modelling only the first three equations were used for North and East axes, while the full set was used for the vertical channel. It was found that the basic set of state-model equations for the Kalman filter had to be augmented by 10 states to include all significant terms describing the behaviour of gyro drifts.

The accelerometers are modelled in similar way, the total error Δa consisting of the bias term ∇, random ramp a^r, two correlated noise terms b^{c_1}, b^{c_2} and a white noise ϑ. There was a significant level of white noise, since that term accounted for the instrument error and for the signal-transmission error, so the model was formally written as :

$$\epsilon_a = \nabla + b^r + b^{c_1} + b^{c_2} + \vartheta \tag{103}$$

The model describing the dynamic behaviour of the error consists of 5 first-order differential equations. Consequently, the state vector used by Kortum in his system model for misalignment (cf. formula (95)) was augmented to 23 components. Kortum discusses also the problem of order reduction following the analysis of covariance of the full order model. The model reduction follows the elimination of the states (and their dynamic equations) which do not influence the behaviour of the system in a significant way, and those which are not observable. The resulting reduced order models are discussed in more detail in 3.5.

In the case of the simplest modelling (cf. [13]) gyro drifts can be represented by a constant, so the corresponding model is

$$\dot{\epsilon}_i = 0 \tag{104}$$

Another option is to represent them either by a random walk in which case

$$\dot{\epsilon}_i = \vartheta_i \qquad \vartheta_i \sim N(0, Q_i) \tag{105}$$

or by first order Markov processes, so

$$\dot{\epsilon}_i = \beta_i \epsilon + \vartheta_i \tag{106}$$

where β_i are suitably chosen coefficients.

The accelerometer biases ∇ may also be modelled by any of the equations (104)-(106). If the initial assumption on the aircraft's INS is to be kept (i.e. the information supplied from aircraft via interface is not corrupted by any error) then the models discussed in 3.2. and 3.3 can be further augmented by inclusion of 3 states describing the behaviour of strapdown gyro drifts and 3 states modelling accelerometer biases. If the assumption is removed, then we have to augment the models by 6 and 12 states respectively. Table 1 summarises the modelling assumptions and resulting dimensions of state models for the linear Kalman filtering applied to the TA using angular rate matching techniques.

	Dimensions of KF (state models) for angular rate matching	
	Schneider's model	Harris-Wakefield model
Rigid structure	3	-
Flexible structure	9	3
(+) Gyro drifts on missile	12	6
(+) Drifts on INS (aircraft) gyros	15	9
(+) Accelerometers biases	18-21	12-15

Table 2. Dimensions of KF - comparison of algorithms.

Inclusion of bias terms into the state vectors has an obvious result in the increased dimensions of the state matrices, which contribute to computational problems (matrix $[H_k(\Phi_{k-1}P^+_{k-1}\Phi^T_{k-1} + \Gamma Q_{k-1}\Gamma^T) + R_k]^{-1}$, which is part of an explicit expression for the Kalman gain, must be found for each step of the procedure). An interesting example of incorporation of bias estimation into the KF was given by Ignani [19]. Two separate, decoupled Kalman filters were designed, one for each set of states (one set describes a linearised kinematic or dynamic model, while the other deals with slowly changing biases). It is assumed here that biases states, although not strictly constant, will undergo only limited variation in time.

The model under consideration is designed using the following assumptions :

- the $N \times 1$ state vector $[x]$ is formed from N_1 and N_2 dimensional vectors x^1, x^2 ($N_1 + N_2 = N$) such that $[x] = [x^1, x^2]$

- $[x^1]_k$ and $[x^2]_k$ represent the dynamic states (angular rates, bending angles, total misalignment angles etc. - depending on the choice of technique) and the bias states (gyro and/or accelerometer) respectively, at the k-th update point,

- system and observation noise parameters satisfy the usual assumption of linear KF (see chapt. 1), with known power spectral densities.

- some of the dynamic states and biases are correlated; this dependency is expressed in the form

$$x_0 = x_0^* + M_0 x^2 \tag{107}$$

where the star refers to those states which are not correlated with the bias and M_0 is a known matrix

- the initial error covariance relationships are

$$P_{x_0} = P^*_{x_0} + M_0 P_{x_0^2} M_0^T \tag{108}$$
$$P_{xx_0^2} = M_0 P_{x_0^2} \tag{109}$$

where P_{x_0}, $P^*_{x_0}$, $P_{x_0^2}$ are the covariance matrices of initial estimation errors of x_0, x_0^* and x_0^2.

Thus the model of Ignani [19] may be presented (using somewhat different notations) in the following way. The system model is

$$x_k^1 = A_{k-1}x_{k-1}^1 + B_{k-1}x_{k-1}^2 + \vartheta_{k-1}^1 \tag{110}$$
$$x_k^2 = x_{k-1}^2 + \vartheta^2 \tag{111}$$

while the measurement model at the n-th update point is

$$z_k = L_k x_k^1 + C_k x_k^2 + \varrho \tag{112}$$

The Kalman filter matrices may now be presented in the following way

$$\Phi = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \qquad H = [\,L \quad C\,]$$
$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \qquad Q = \begin{bmatrix} Q_{x^1} & 0_{N_2} \\ 0_{N_1} & Q_{x^2} \end{bmatrix}$$

while the gains matrix will be partitioned in the form $K = [K^1,\ K^2]^T$. Using these relationships we have

$$\begin{bmatrix} \hat{x}^1 \\ \hat{x}^2 \end{bmatrix}_k^- = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix}_{k-1} \begin{bmatrix} \hat{x}^1 \\ \hat{x}^2 \end{bmatrix}_{k-1}^+ \tag{113}$$

$$P_k^- = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}_k^- = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix}_{k-1} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}_{k-1}^+ \begin{bmatrix} A & B \\ 0 & I \end{bmatrix}_{k-1}^T + \begin{bmatrix} Q_{x^1} & 0_{N_2} \\ 0_{N_1} & Q_{x^2} \end{bmatrix}_{k-1} \tag{114}$$

$$\begin{bmatrix} K^1 \\ K^2 \end{bmatrix}_k = P_k^- \begin{bmatrix} L \\ C \end{bmatrix}_k \left\{ [L\ C]_k P_k^- \begin{bmatrix} L \\ C \end{bmatrix}_k + R_k \right\}^{-1} \tag{115}$$

$$P_k^+ = \left\{ I - \begin{bmatrix} K^1 \\ K^2 \end{bmatrix}_k \begin{bmatrix} L \\ C \end{bmatrix}_k \right\} P_k^- \tag{116}$$

$$\begin{bmatrix} \hat{x}^1 \\ \hat{x}^2 \end{bmatrix}_k^+ = \begin{bmatrix} \hat{x}^1 \\ \hat{x}^2 \end{bmatrix}_k^- + K_k \left\{ z_k - [L \quad C] \begin{bmatrix} \hat{x}^1 \\ \hat{x}^2 \end{bmatrix}_k^- \right\} \tag{117}$$

The block structure of system, observation and variance matrices leads to the design of two independent estimators for the dynamics states and biases vector. The formulae are given explicitly in [19].

3.4 Velocity and position matching.

A separate set of methods are those relying on velocity information and not directly utilising information on angular rates (in a sense that the difference of the rates enters neither the system nor observation vector, but it may be used as an entry in the system or observation matrix). If the above condition is not fulfilled and differences in velocities as well as the differences of angular rates enter the system and/or observation vector, we shall refer to the resulting models as augmented, or "combined".

Velocity and position matching methods for transfer alignment have been discussed in numerous publications - for more details see - [1]-[4], [8], [18]-[26]. Only standard approaches and models will be discussed in detail.

Farrell in [26] proposed an algorithm for TA based on the 9-dimensional system model with state-variables vector formed from velocity error, misalignment angle and drift components in the from $x = [\Delta v,\ \gamma,\ \epsilon]$. Physical relations leading to the model stem from an the extension of discussion given in previous paragraphs. For the system model we have

$$\Delta\dot{\mathrm{v}} = \zeta \times f + \vartheta_a \tag{118}$$

$$\dot{\zeta} = \epsilon + \vartheta_\omega \tag{119}$$

$$\dot{\epsilon} = -1/\tau\epsilon + \vartheta_\alpha \tag{120}$$

while the observation model may be written as

$$z = \begin{bmatrix} \Delta v_x^* \\ \Delta v_y^* \\ \Delta v_z^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta v_x \\ \Delta v_y \\ \Delta v_z \\ \zeta_x \\ \zeta_y \\ \zeta_z \\ \epsilon_x \\ \epsilon_y \\ \epsilon_z \end{bmatrix} + \begin{bmatrix} \varrho_x \\ \varrho_y \\ \varrho_z \end{bmatrix} \tag{121}$$

The complete transfer alignment algorithm with flexibility taken into account is based on 15 states vector for the system model (9 states from the last model plus 6 first order equations modelling the flexibility, cf. earlier approach by Schneider - described in 3.2). A simulation block diagram for the proposed algorithm is presented in Figure 4.

Bar-Ithzack [25] developed a full 9-state filter with the state vector consisting of 3 position errors, corresponding North, East and down velocity errors components and the misalignment errors. He then analysed the possible order reduction by experimenting with 7- and 5-state filters, then further reducing the order with a set of 3-, and 2-state time-sharing filters. Some of these results are discussed in the next paragraph.

The vector of states of Bar-Itzhack's 9-state model can be written as :

$$x = [r_N,\ r_E,\ r_D,\ \Delta v_N,\ \Delta v_E,\ \Delta v_D,\ \gamma_N,\ \gamma_E,\ \gamma_D]^T \tag{122}$$

so the (continuous), state-space system model may be represented by the equation

$$\dot{x} = \Phi x + \vartheta \tag{123}$$

where the matrix Φ is defined in the following way

$$\begin{bmatrix} 0 & -\dot{\lambda} sL & \dot{L} & 1 & 0 & 0 & 0 & 0 & 0 \\ \dot{\lambda} sL & 0 & \dot{\lambda} cL & 0 & 1 & 0 & 0 & 0 & 0 \\ -\dot{L} & -\dot{\lambda} cL & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \Phi_{14} & \Phi_{24} & 0 & 0 & -ZsL & \dot{L} & 0 & -f_D & f_E \\ 0 & \Phi_{25} & 0 & ZsL & 0 & ZcL & d_D & 0 & -f_N \\ \Phi_{16} & \Phi_{26} & \frac{2g}{R} & -\dot{L} & -ZcL & -C & -f_E & f_N & 0 \\ -\frac{\Omega sL}{R} & \Phi_{27} & \frac{\dot{\lambda} cL}{R} & 0 & R^{-1} & 0 & 0 & -UsL & \dot{L} \\ -\frac{V_E}{R^2} & \frac{\dot{\lambda} sL}{R} & -\frac{\dot{L}}{R} & -R^{-1} & 0 & 0 & UsL & 0 & UcL \\ \Phi_{19} & \Phi_{29} & -\frac{\dot{\lambda} sL}{R} & 0 & -\frac{\tan L}{R} & 0 & -\dot{L} & -UcL & 0 \end{bmatrix} \tag{124}$$

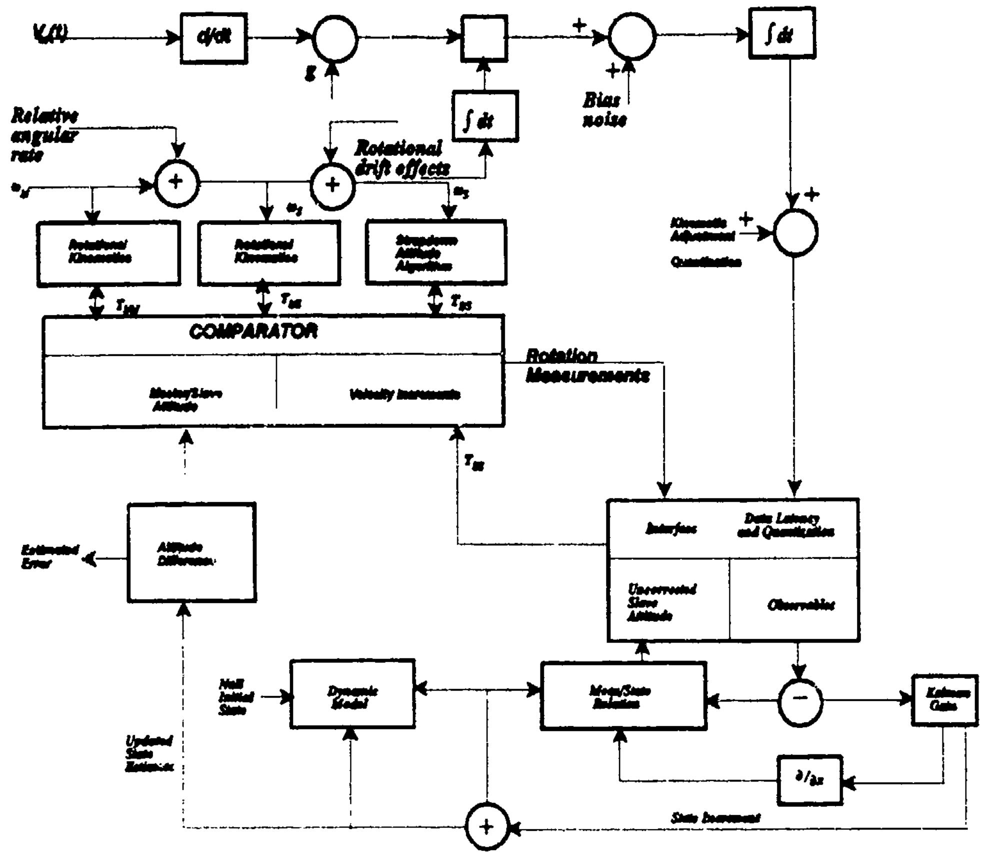

Figure 4 Simulation Block Diagram for TA
Source : Farrell [26]

with

K_d - the damping gain in altimeter damping loop

$$U = \Omega + \dot{\lambda}$$

$$Z = 2\Omega + \dot{\lambda}$$

$$\Phi_{14} = -\frac{f_D + g}{R}$$

$$\Phi_{16} = -\frac{-f_N}{R} + K_d L$$

$$\Phi_{19} = -\left(\frac{\Omega c L}{R} + \frac{\dot{\lambda}}{R c L}\right)$$

$$\Phi_{24} = \frac{f_E \tan L}{R}$$

$$\Phi_{25} = -\frac{f_D + g + f_N \tan L}{R}$$

$$\Phi_{26} = \frac{f_E}{R} + C\dot{\lambda} c L$$

$$\Phi_{27} = \frac{v_D}{R^2} + \frac{\dot{L} \tan L}{R}$$

$$\Phi_{29} = -\frac{\tan L}{R}\left(\dot{L} \tan L + \frac{v_D}{R}\right)$$

while the noise vector ϑ is

$$[0,\ 0,\ 0,\ \nabla,\ \epsilon]^T$$

The "truth model" used in [25] for the comparison of the simulation results includes 24 states, and the performance of the designed filters was analysed using a true covariance simulation program (including the states of "truth model" and the differences between the values of the states produced by the filters and the corresponding values obtained from the "truth model").

Sutherland [27] investigated the applications of KF for transfer alignment using the optimal control techniques to determine manoeuvres which minimise filter errors. He gives the following five-state system model

$$\frac{d}{dt}\begin{bmatrix}\Delta v_x,\\ \Delta v_y\\ \zeta_x\\ \zeta_y\\ \zeta_z\end{bmatrix} = \begin{bmatrix}0 & 0 & 0 & g & a_y\\ 0 & 0 & -g & 0 & -a_x\\ 0 & R^{-1} & 0 & \omega_z & -\omega_y\\ -R^{-1} & 0 & -\omega_z & 0 & \omega_x\\ 0 & \frac{-\tan\lambda}{R} & \omega_y & -\omega_x & 0\end{bmatrix}\begin{bmatrix}\Delta v_x,\\ \Delta v_y\\ \zeta_x\\ \zeta_y\\ \zeta_z\end{bmatrix} + \begin{bmatrix}\nabla_x\\ \nabla_y\\ \epsilon_x\\ \epsilon_y\\ \epsilon_z\end{bmatrix} \tag{125}$$

with observation model defined as

$$\begin{bmatrix}z_1\\ z_2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta v_x,\\ \Delta v_y\\ \zeta_x\\ \zeta_y\\ \zeta_z\end{bmatrix} + \begin{bmatrix}\varrho_1\\ \varrho_2\end{bmatrix} \tag{126}$$

The presence of acceleration terms in the system matrix was instrumental in determining optimal manoeuvres for the alignment. Additional assumptions, which made the application of a linear optimal control technique possible, were : constant speed, chosen maximal value for the terms of horizontal acceleration and known initial rms misalignment errors and other stochastic parameters. The control parameter was chosen to be the heading rate of change, while the performance index minimised the mean square of errors of the misalignment angles at a specified final time. The found solution (as can be expected with linear control problems and the bounds on control parameters specified) indicated the necessity of applying the switching control with maximal rate of change of heading angle (bang-bang solution).

Bryson in [8] designed the following 5-state model (horizontal components of velocity vector and misalignment angle) used for the description of errors of an INS with local level, North-pointing platform :

$$\begin{bmatrix}\Delta\dot v_E\\ \Delta\dot v_N\\ \dot\gamma_U\\ \dot\gamma_E\\ \dot\gamma_N\end{bmatrix} = \begin{bmatrix}0 & (2\Omega_U+\rho_U) & f_N & 0 & -f_U\\ -(2\Omega_U+\rho_U) & 0 & -f_E & f_U & 0\\ 0 & 0 & 0 & (\Omega_N+\rho_N) & -\rho_E\\ 0 & 0 & -(\Omega_N+\rho_N) & 0 & (\Omega_U+\rho_U)\\ 0 & 0 & \rho_E & -(\Omega_U+\rho_U) & 0\end{bmatrix}\begin{bmatrix}\Delta v_E\\ \Delta v_N\\ \gamma_U\\ \gamma_E\\ \gamma_N\end{bmatrix} + \begin{bmatrix}-g^2/R^2\delta R_E + n_E\\ -g^2/R^2\delta R_N + n_N\\ \epsilon_U\\ \epsilon_E\\ \epsilon_N\end{bmatrix} \tag{127}$$

where

$$(\rho_U,\ \rho_E,\ \rho_N) = (\frac{v_E}{R}\tan\lambda,\ -\frac{v_N}{R},\ \frac{v_E}{R})$$

The model was used to derive a simple formula for identification of misalignment as a function of the flight profile. For the straight and level flight the "fast observer" for East-West tilt was expressed as

$$\begin{aligned} \dot{\gamma}_E^* &= \frac{1}{\tau}\left[(2\Omega_U + \rho_U)\frac{\Delta v_E}{g} + \frac{\Delta r_N}{R} - \hat{\gamma}_E\right], \\ \hat{\gamma}_E &= \gamma_E^* + \frac{1}{\tau}\frac{\Delta v_N}{g}, \qquad \hat{\gamma}_E(0) = 0 \end{aligned} \tag{128}$$

while for the heading error the equations for fast observer are

$$\begin{aligned} \dot{\gamma}_N^* &= \frac{1}{\tau}\left[(2\Omega_U + \rho_U)\frac{\Delta v_N}{g} - \frac{\Delta r_N}{R} - \hat{\gamma}_N\right], \\ \hat{\gamma}_N &= \gamma_N^* - \frac{1}{\tau}\frac{\Delta v_E}{g}, \qquad \hat{\gamma}_N(0) = 0 \end{aligned} \tag{129}$$

Bryson and Powell derived also the fast observers for misalignment during manoeuvring flight (constant-altitude, constant heading deceleration-acceleration and constant bank angle turn). Details are contained in [8].

Baziw and Leondes in [18] presented a number of different designs for the Kalman filter based on velocity and position errors (being first and second integrals of the accelerometer outputs) for the observation model. It initially includes the errors due to scale factor, biases and a correlated noise modelled by Markov processes, although it may be reduced to just position and velocity errors (+ white noise).

The state vectors (for a number of the system models presented) include some of the following variables : positions and velocities, random vibrations modelled by second order Markov processes, coloured noise, misalignments, biases, drifts, master INS error parameters, observation error parameters, and the error vectors of the nominal positions of the master and slave guidance instruments. The dimension of the state vectors varies from 6 to 51, the preferred option consisting of 36 variables. Baziw and Leondes also discussed the possibility of including the angular velocity observations into the model.

Yamamoto and Brown [2] described an algorithm for alignment of an inertially guided air-to-surface strategic missile based on position matching (latitude, longitude) for the measurement model and a 10-state system model. The state vector includes two position errors, horizontal velocity errors, three misalignment angles and three states of gyro drifts, so using our notation $\mathbf{x} = [r_N, r_E, \Delta v_N, \Delta v_E, \gamma_N, \gamma_E, \gamma_D, \epsilon_N, \epsilon_E, \epsilon_D]^T$, the measurement model can be written as :

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \Gamma \varrho \tag{130}$$

The proposed filter was examined during a complex simulation analysis. Captive flight of missiles has been simulated, together with a separate assessment of the Kalman filter for the carrier and for the missile. Since the lower limit of the missile's alignment error depends on the accuracy of the data supplied from the carrier's INS, prior to the data transfer the carrier's inertial navigator is aligned (from cold-start conditions) combining the data from the doppler radar and position fix radar. The simulation process described in [2] is presented in Figure 5.

Kain and Cloutier [28] presented an analysis of 24 state, 6 measurement Kalman filter implementable on 25 mHz M68020 microprocessor with M68882 coprocessor. Using this particular

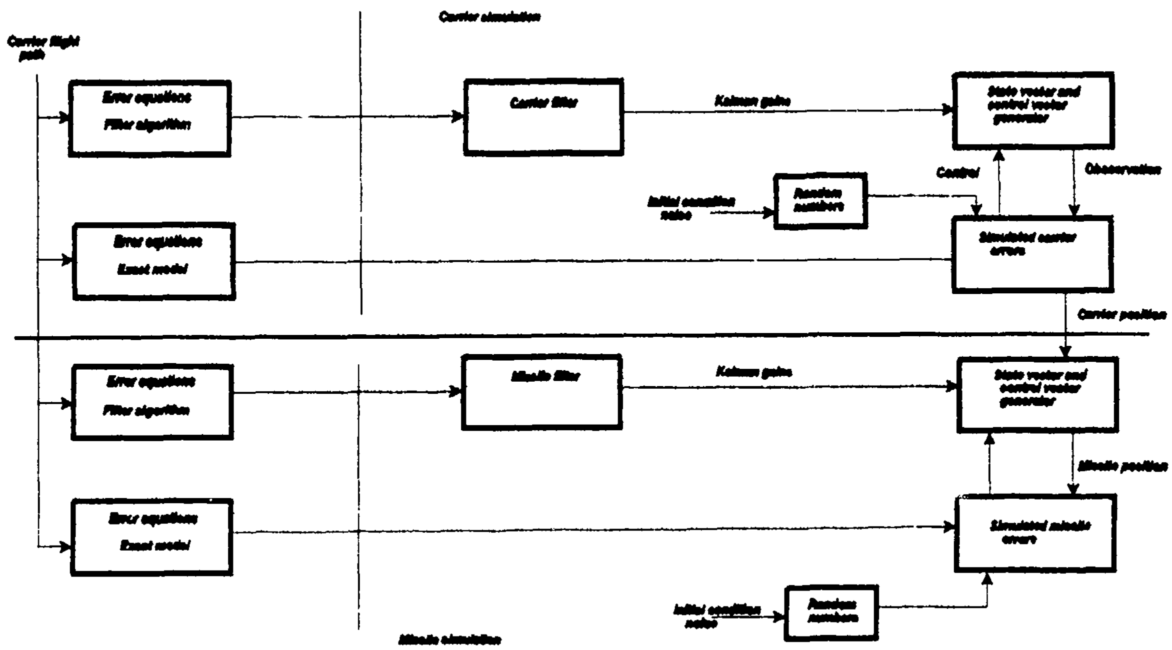

Figure 5 Simulation of TA for SRAM missile
Source : Yamamoto and Brown [2].

harware configuration the propagation and update cycle time of 28 milliseconds was achieved. The states include velocity erros, angular differences, physical alignment errors, accelerometer and gyro biases and scale factor errors, and gyro g-sensitive drifts. The Monte Carlo simulation was performed for the truth-model consisting of 42 states (24 states for the filter and 6 third order Markov processes for the wing flexure modelling). The misalignment error was less than 1 mrad per axis and the convergence time less than 10 seconds.

3.5 Reduced order filters.

The models derived by Bar-Itzhack (cf. [25]) have been used for in-flight alignment (five states model) and transfer alignment (seven states model), the latter including information on position error. The filter based on reduced order models is designed for relatively fast (approximately 3 minutes) alignment of aircraft' INS. The first two second-order filters (presented later as A and B, see equations (133)-(136)) describe velocity error in straight and level flight. The third order filter C modelled by equation (137) describes the behaviour of velocity error during the horizontal (S-shaped) manoeuvre. The reduced order filters operate in time-sharing mode thus saving computational effort.

The filters operate in sequence and their operation has been synchronised with the flight profile. During about 30 seconds of straight and level flight filters A and B operate in an alternate way, with no filter being "frozen". When one of the two filters does propagation and update i.e full KF operates (equations (1)-(9)), the state vector of the other is just being propagated (equation (5)), which only requires a knowledge of entries of the state matrix and the state vector calculated in the preceding time interval. After that period the estimated value of North and East misalignment angles are removed from the output of inertial sensors (which is referred to as resetting control) and the third order filter C estimates γ_D. This part of the alignment was performed in about 3 minutes (simulation results quoted by Bar-Itzhack show convergence for a lateral S-shaped manoeuvre lasting about 210 seconds).

The five state model chosen in [25] for in-flight alignment is

$$\frac{d}{dt}\begin{bmatrix}\Delta v_N\\ \Delta v_E\\ \gamma_N\\ \gamma_E\\ \gamma_D\end{bmatrix} = \begin{bmatrix}0 & 2\Omega_D & 0 & g & f_E\\ -2\Omega_D & 0 & -g & 0 & -f_N\\ 0 & R^{-1} & 0 & \Omega_D & 0\\ -R^{-1} & 0 & -\Omega_D & 0 & \Omega_N\\ 0 & -\frac{\tan L}{R} & 0 & -\Omega_N & 0\end{bmatrix}\begin{bmatrix}\Delta v_N\\ \Delta v_E\\ \gamma_N\\ \gamma_E\\ \gamma_D\end{bmatrix} + \begin{bmatrix}\vartheta_1\\ \vartheta_2\\ \vartheta_3\\ \vartheta_4\\ \vartheta_5\end{bmatrix} \tag{131}$$

with corresponding observation model

$$\begin{bmatrix}z_1\\ z_2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta v_N\\ \Delta v_E\\ \gamma_N\\ \gamma_E\\ \gamma_D\end{bmatrix} + \begin{bmatrix}n_1\\ n_2\end{bmatrix} \tag{132}$$

The seven-state model for transfer alignment includes two horizontal position errors (r_N and r_E). Time sharing reduced-order filters were designed in the following way :

Filter A

$$\frac{d}{dt}\begin{bmatrix}\Delta v_N\\ \gamma_E\end{bmatrix} = \begin{bmatrix}0 & g\\ -R^{-1} & 0\end{bmatrix}\begin{bmatrix}\Delta v_N\\ \gamma_E\end{bmatrix} + \begin{bmatrix}\vartheta_{vN}\\ \vartheta_{\gamma E}\end{bmatrix} \tag{133}$$

$$z_1 = [1\ 0]\begin{bmatrix}\Delta v_N\\ \gamma_E\end{bmatrix} + n_1 \tag{134}$$

Filter B

$$\frac{d}{dt}\begin{bmatrix}\Delta v_E\\ \gamma_N\end{bmatrix} = \begin{bmatrix}0 & -g\\ R^{-1} & 0\end{bmatrix}\begin{bmatrix}\Delta v_E\\ \gamma_N\end{bmatrix} + \begin{bmatrix}\vartheta_{vE}\\ \vartheta_{\gamma N}\end{bmatrix} \tag{135}$$

$$z_1 = [1\ 0]\begin{bmatrix}v_E\\ \gamma_N\end{bmatrix} + n_2 \tag{136}$$

Filter C

$$\frac{d}{dt}\begin{bmatrix}\Delta v_N\\ \Delta v_E\\ \gamma_D\end{bmatrix} = \begin{bmatrix}0 & 2\Omega_N & f_E\\ -2\omega_N & 0 & -f_N\\ 0 & -\frac{\tan L}{R} & 0\end{bmatrix}\begin{bmatrix}\Delta v_N\\ \Delta v_E\\ \gamma_D\end{bmatrix} + \begin{bmatrix}\vartheta_{vN}\\ \vartheta_{vE}\\ \vartheta_{\gamma D}\end{bmatrix} \tag{137}$$

$$\begin{bmatrix}z_1\\ z_2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta v_N\\ \Delta v_E\\ \gamma_D\end{bmatrix} + \begin{bmatrix}n_1\\ n_2\end{bmatrix} \tag{138}$$

Using the approximate relation i.e.

$$R^{-1} \approx 0$$
$$\Omega_N \approx 0$$

[11] also gives the following simplified models :

Filter A

$$\frac{d}{dt}\begin{bmatrix}\Delta v_N\\ \gamma_E\end{bmatrix} = \begin{bmatrix}0 & g\\ 0 & 0\end{bmatrix}\begin{bmatrix}\Delta v_N\\ \gamma_E\end{bmatrix} + \begin{bmatrix}\vartheta_{vN}\\ \vartheta_{\gamma E}\end{bmatrix} \tag{139}$$

Filter B

$$\frac{d}{dt}\begin{bmatrix}\Delta v_E \\ \gamma_N\end{bmatrix} = \begin{bmatrix}0 & -g \\ 0 & 0\end{bmatrix}\begin{bmatrix}\Delta v_E \\ \gamma_N\end{bmatrix} + \begin{bmatrix}\vartheta_{vE} \\ \vartheta_{\gamma N}\end{bmatrix} \tag{140}$$

Filter C

$$\frac{d}{dt}\begin{bmatrix}\Delta v_N \\ \Delta v_E \\ \gamma_D\end{bmatrix} = \begin{bmatrix}0 & 0 & f_E \\ 0 & 0 & -f_N \\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta v_N \\ \Delta v_E \\ \gamma_D\end{bmatrix} + \begin{bmatrix}\vartheta_{vN} \\ \vartheta_{vE} \\ \vartheta_{\gamma_D}\end{bmatrix} \tag{141}$$

The results of the simulation studies quoted in [25] confirm the adequacy of simplified filters (their performance degraded by less than 2% than filters with no approximation introduced in the state matrices).

Kortum [7] investigated possible reductions in the order of the Kalman filter using covariance analysis. The results from the simulation of the full-size 15-states model were compared with lower order models (ranging from 3 states to 7). The following conclusions were reached. When the 7-order filter provided the bounds for accuracy, reduction of the order to 4 states did not influence the accuracy of estimates of the remaining states in the short period of time. For the time interval of 10 minutes the only noticeable effect concerns the influence of gyro drift about the North-South axis and its influence on γ_N. Reduction of the order to 3 states only was not recommended (the accuracy of the estimate of γ_N was appreciably affected without the inclusion of the estimate of the corresponding drift rate).

Note that all the methods of this section require knowledge of velocity and/or position. Thus, although they may be superior to angular rate matching methods, they are not applicable to attitude and heading reference systems which do not compute the full navigation solution.

4. CONVERGENCE OF FILTERS AND COMPUTATIONAL REQUIREMENTS

Divergence of the Kalman filter produces growing estimate errors, which are larger than theoretically predicted. It is generally caused by mismatched or uncontrollable ("undisturbable") system models. Controllability/observability conditions should be applied to the derived models before proceeding with the simulation. Among the methods for preventing divergence are pole shifting, eigenvalue constraints and added noise (see [5] and [29] for details). Another problem arises in an efficient implementation of a particular form of Kalman filter (viz. usage of Joseph form for covariance matrix and square root filtering in order to maintain properties of the covariance matrix).

Mendel [30] analysed the computational problems associated with the implementation of code for KF algorithm. The following table summarises the requirement in terms of number of multiplications and additions needed for KF where N is the dimension of the state vector of the system model, while M is the dimension of the state vector of the measurement model.

Variable	Additions	Multiplications
State variables extrapolation	N^2	$N^2 + N$
Covariance matrix extrapolation	$4N^3 - 3N^2$	$4N^3$
Kalman gain	$NM(2M + N - 2) + M^3$	$M(M^2 + N^2 + 2MN)$
State update	$2MN$	$2MN$
Covariance update	$M^3 + N^2(M - 1)$	$N^3 + N^2M$

Table 2. Computational requirement of KF.
Source : Mendel [30].

Assuming that 5 μsec is an upper limit for addition or multiplication Table 3 compares the execution times for some of the presented methods for transfer alignment.

Method	N	M	Time (1 cycle) in milliseconds
Rate matching :			
Schneider A	3	3	3
Schneider B	9	3	40
Rate and acceleration matching :			
Schneider B	9	6	50
Harris-Wakefield	3	3	3
Velocity matching :			
Farrell	9	3	40
Bar-Itzhack	7	3	20
Bar-Itzhack	3	2	2
Bar-Itzhack	2	1	0.5
Position matching :			
Yamamoto-Brown	10	2	50

Table 3. Time requirement for TA computation

The above numbers should be treated as an approximation of the upper limit of the time needed for one cycle of calculations assuming that both additions and multiplications take 5 μsec. The time needed for calculations may be further reduced if the philosophy of the design follows some general rules. These include (cf. [5] for details) :

(1) Reduction of the state dimensions while maintaining the basic characteristics of the system.

(2) Simplification (i.e. linearisation) of the model by neglecting (or replacing by the linear term of an expansion) the nonlinear terms, couplings etc.

(3) Using the symmetry of matrices, sparsity, and block-decomposition method.

(4) Using precompute values, known approximations and prefiltering (viz. compression of data) to make sample periods longer.

(5) Implementation of a "square root filter" as a way to remove the double precision requirement.

(6) Approximation of stored constants as powers so multiplications are replaced by shift operations.

(7) Efficient programming practices.

The order of the Kalman filter should be considered in view of the time allowed for the alignment process. The shorter the time allowed for the process the fewer the number of variables to be included in the state vectors of the system/measurement models.

Systematic design procedure of a Kalman filter for the transfer alignment problem should consists of a number of steps (cf. [5] for a general case) :

(1) Development of a mathematical model for the transfer alignment. At this stage it may be necessary to include the models of both inertial guidance instruments (master and slave INSs) and possibly an interface if it is known to contribute to time lags. Flight simulation (6DOF) using the complete model usually follows and is validated with experimental data. Error analysis then leads to any necessary changes in the models.

(2) Generation of full order Kalman filter followed by a covariance analysis.

(3) Reduction of order by removing some of the variables especially cross-coupled terms, employing approximations, possibly deleting uncoupled noncontributing equations. It may lead to a number of models. Design a proper KF should include an analysis of each model.

(4) Covariance performance analysis for each reduced order filter. Each of the models is to be "fine tuned", which in particular includes the choice of initial values of variance of covariance matrices.

(5) Monte Carlo simulation and analysis of the best designs from the previous step.

(6) Selection of the design based on analysis of the performance versus computing efficiency (real-time requirements).

(7) Implementation and operational tests.

5. CONCLUSIONS

Presented methods differ with respect to computer memory required, conditions for convergence and consequently the time of alignment process. Combined rate and acceleration matching achieved convergence in less than 10 seconds (during the lateral manoeuvre), while the rate matching alone required the time of order of 1 minute. Velocity matching methods (see 3.6-3.7) required 3-5 minutes (with the exception of the method reported in [26]), while position matching algorithms developed for aided INS (cf. [64]) needed 20 minutes to attain a steady-state. However, these numbers should only serve as a very general indication, since the conditions of experiments varied. Another factor is the computational time required per cycle of Kalman filter algorithm, which depends on the number of system and observation variables. Given the same hardware configuration it ranges from 0.5 milliseconds - in case of 2 system variables and 1 observation variable needed in one velocity matching method - to 50 milliseconds in case of 10 system variables and 2 observation variables (position matching method).

ACKNOWLEGMENTS

The authors wish to thank Dr. R. Miller of Flight Management, Messrs. K.H. Lloyd, D.J. Hards of Guidance, Control and Simulation and Dr. D. Brown of Dynamics and Trials for many valuable discussions held during the preparation of this report.

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Inertial navigation Inertial measurement and units Calibration Guided weapons	1606

16. ABSTRACT

This report presents methods for transfer alignment (TA) of inertial navigation systems (INS) which have been published in the accessible literature during the last three decades. Kalman filtering techniques based on linearised dynamics dominated in the literature of the subject. Methods of TA can be classified as angular rates, velocity or position matching. In each case a number of assumptions are made to ensure the validity of proposed technique. The accuracy of filters depends on the particular implementation viz. allowable manoeuvres and time of TA, microprocessor used, vibration environment, inclusion of wing flexure into the model, type of application under discussion (range of missile or time of flight), quality of output from inertial measurement units, etc. Authors briefly discuss methods of in-flight transfer alignment of INS taking into account these assumptions.

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17. IMPRINT **AERONAUTICAL RESEARCH LABORATORY, MELBOURNE**		
18. DOCUMENT SERIES AND NUMBER Guided Weapons Technical Memorandum 012	19. COST CODE 409 488	20. TYPE OF REPORT AND PERIOD COVERED
21. COMPUTER PROGRAMS USED		
22. ESTABLISHMENT FILE REF.(S)		
23. ADDITIONAL INFORMATION (AS REQUIRED)		